OPTIMIZATION OF THE BOX SECTION OF THE MAIN GIRDER OF THE DOUBLE BEAM BRIDGE CRANE ACCORDING TO THE CRITERIA OF LATERAL STABILITY AND LOCAL STABILITY OF PLATES

Goran PAVLOVIC² - Mile SAVKOVIC¹ *, - Milomir GASIC¹ - Radovan BULATOVIC¹ - Nebojsa ZDRAVKOVIC¹

¹ University of Kragujevac, Faculty of Mechanical and Civil Engineering in Kraljevo, Serbia
² Knežinje Milice 26/27, 37240 Trstenik, Serbia

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Abstract: The paper considers the problem of optimization of the box section of the main girder of the double beam bridge crane for the ease of placing the rail above the web plate. Reduction of the girder mass is set as the objective function. Research on the mutual dependence of geometric parameters of cross section, and their ratio, using the method of Lagrange multiplier as characteristic criteria is done. Also, the paper shows an algorithm for selection of the optimal criteria and determining the optimal geometric parameters of cross section. The criteria of lateral stability and local stability of plates were applied as the constraint functions. The obtained results of optimization of geometrical parameters were verified on numerical examples.

Key words: box section, double beam bridge crane, optimization, lateral stability, local stability of plates

1. INTRODUCTION

The goal in the process of designing the carrying structure of the double beam bridge crane is determination of optimum dimensions of rectangular box cross-section of the main girder. The main girder is the most responsible part of the double beam bridge crane and therefore it is necessary, during optimization, to affect the increase in its carrying capacity with simultaneous reduction of its mass. The mass of the main girder has the largest share in the total mass of the bridge crane.

That is the reason why the selection of the optimum shape and geometrical parameters, which influence the reduction of mass and costs of manufacturing, is the subject of research of many authors regardless of whether they deal specifically with cranes or carrying structures in general ([1-14]). Most authors set permissible stress or two constraint functions: permissible stress and permissible deflection as the constraint function. The criterion of lateral stability has lately been increasingly applied as the constraint function ([7], [11], [12] and [13]). Having in mind all the above mentioned results and conclusions, the aim of this paper is to define optimum values of rectangular box cross-section of the main girder that will lead to the reduction of its mass.

2. MATHEMATICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area. Minimization of the mass corresponds to minimization of the volume, i.e. the area of the cross section of the girder, where the given boundary conditions must be satisfied. The area of the cross section primarily depends on: height and width of the girder, thickness of plates and their mutual relations.

The optimization problem defined in this way can be given the following general mathematical formulation: minimize $f(X)$ subject to $g(X) \leq 0$, where:

- $f(X)$ the objective function,
- $g(X) \leq 0$ the constraint function,

$X = \{x_1, ..., x_D\}$ represents the design vector made of $D$ design variables. Design variables are the values that should be defined during the optimization procedure. In this paper optimization for the criterion of lateral stability (3.1) and the criterion of local stability of plates was performed (3.2):

$g_1 = \sigma_{x_1} - \sigma_{x_2} \leq 0 \quad (1.1)$

$g_2 = \sigma_{x_2} - \sigma_{x_3} \leq 0 \quad (1.2)$

where:

- $\sigma_{x_1}, \sigma_{x_2}$ - the calculation and permissible stresses in lateral buckling of the girder,
- $\sigma_{x_2}, \sigma_{x_3}$ - the calculation and permissible stresses local stability of plates.

The Lagrange function is defined in the following way:

$\Phi = A + \lambda_1 \cdot g_1 + \lambda_2 \cdot g_2 \quad (2)$

and

$\frac{\partial \Phi}{\partial \lambda_1} = 0 \Rightarrow g_1 = 0 \quad (3.1)$

$\frac{\partial \Phi}{\partial \lambda_2} = 0 \Rightarrow g_2 = 0 \quad (3.2)$

The paper considers the problem of optimization of the box section of the main girder of the double beam bridge crane for the ease of placing the rail above the web plate. Reduction of the girder mass is set as the objective function. Research on the mutual dependence of geometric parameters of cross section, and their ratio, using the method of Lagrange multiplier as characteristic criteria is done. Also, the paper shows an algorithm for selection of the optimal criteria and determining the optimal geometric parameters of cross section. The criteria of lateral stability and local stability of plates were applied as the constraint functions. The obtained results of optimization of geometrical parameters were verified on numerical examples.

*Correspondence Author’s Address: University of Kragujevac, Faculty of Mechanical and Civil Engineering, Dositejeva 19, 36000 Kraljevo, Serbia, savkovic.m@mfkv.kg.ac.rs
\[
\lambda_1 \left( \frac{\partial g_1}{\partial b} \frac{\partial A}{\partial h} + \frac{\partial g_1}{\partial h} \frac{\partial A}{\partial b} \right) + \lambda_2 \left( \frac{\partial g_2}{\partial b} \frac{\partial A}{\partial h} + \frac{\partial g_2}{\partial h} \frac{\partial A}{\partial b} \right) = 0
\]

Since \( \lambda_1, \lambda_2 \neq 0 \), it is obtained:

1. \[
\frac{\partial A}{\partial b} \frac{\partial g_1}{\partial h} = \frac{\partial A}{\partial h} \frac{\partial g_1}{\partial b} \quad \text{and} \quad g_1 = 0
\]

2. \[
\frac{\partial A}{\partial b} \frac{\partial g_2}{\partial h} = \frac{\partial A}{\partial h} \frac{\partial g_2}{\partial b} \quad \text{and} \quad g_2 = 0
\]

3. OBJECTIVE AND CONSTRAINT FUNCTIONS

3.1. Objective function

The objective function is represented by the area of the cross section of the box girder. The paper treats two optimization parameters \((h, b)\). The wall thicknesses \(t_1\) and \(t_2\) (Fig. 1) are not treated as optimization parameters for the purpose of simplification of the procedure. Their values were adopted in accordance with the recommendations of crane manufacturers [14].

The vector of the given parameters is:

\[
\bar{x} = (M_{cv}, M_{ch}, Q, L, G_k, E, k_a, \ldots)
\]

where:

- \(M_{cv}\) and \(M_{ch}\) are the bending moments in the vertical and horizontal planes,
- \(Q\) – the carrying capacity of the crane,
- \(L\) - the span of the crane,
- \(G_k\) - the mass of the crane cab,
- \(E\) - the module of elasticity of the main girder of the crane,
- \(k_a\) – the dynamic coefficient of crane load in the horizontal plane [10].

**Fig.1. The box section of the main girder of the bridge crane**

The area of the cross section, i.e. the objective function, is:

\[
A(h, b) = f(h, b) = 2 \cdot (e \cdot b \cdot h + h^3) / s ,
\]

where:

- \(e = t_1 / t_2\) - the ratio between thicknesses of plates at the flange and at the web,
- \(s = h / t_2\) - the ratio between the height and thickness of the plate at the web.

Using the fact that \(s \gg e\) and \(s \ll k\) the coefficients with the form \(\beta_s\) and \(\alpha_s\) can be simplified:
\[ \beta_x \approx \frac{1}{2} \sqrt{\frac{k + 3 - e}{3(e + k)}} \quad \alpha_x \approx \frac{k + 3 - e}{6(e + k)} . \]  

This approximation can be graphically represented (Figure 2), where it is seen that deviations are negligible in the considered range of parameters \( k \).

By repeating the procedure for the moment of inertia and the moment of resistance for the \( y \) - axis, the following values of coefficients are obtained:

\[ \beta_y \approx \frac{1}{2} \sqrt{\frac{3k \cdot f^2 + e}{3(e + k)}} \quad \alpha_y \approx \frac{3k \cdot f^2 + e}{6(e + k)} . \]  

### 3.2. Constraint functions

#### 3.2.1. The criterion of lateral stability

Testing of the box girder stability against lateral buckling was carried out in compliance with the Serbian standards of the group [8]. In accordance with the standards [8], the compression zone of the box girder is observed as an independent bar which is controlled against buckling due to the equivalent force arising from the bending moment of the girder (Figure 3).

The pressure force \( D \) of the imagined bar acts at point "o" whose section is the hatched contour (\( A_y \)). The pressure force along the girder is variable, and the defined length of buckling is (\([9],[10]):\( l = 0,63 \cdot L \), whereas the equivalent pressure force along the girder is:

\[ D = \left\{ \sigma_y \cdot dA = \frac{M_{sy} \cdot S_{sy}}{l_y} \right\}. \]  

This criterion is fulfilled if the condition of lateral stability is satisfied:

\[ g_i = \sigma_{si} - \sigma_{si} = \frac{D}{A_y \cdot x} + 0,9 \cdot \frac{M_{sy}}{W_y} \leq 0, \]  

where:

\[ \sigma_{si} = 0,76 \cdot f_y \] - critical stress, \( f_y \) - yield strength.

The buckling coefficient \( \chi \) has the following values [10]:

\[ \chi = 1, \] if the relative slenderness of the bar is \( \lambda \leq 0,2 \), i.e.

\[ \chi = \frac{2}{\beta + \sqrt{\beta^2 - 4 \cdot \lambda}} . \]  

if the relative slenderness of the bar is \( \lambda > 0,2 \) (according to recommendations it is always greater than 0,2). For the box cross section of the main girder of the crane, the coefficient \( \beta \) is:

\[ \beta = 1 + 0,489 \cdot (\lambda - 0,2) + \lambda^2 . \]  

The relative slenderness of the bar (\( \lambda \)) is calculated by the expression:

\[ \lambda = \frac{0,63 \cdot L}{\beta \cdot b \cdot \lambda} , \]  

which, after some transformation, can be written as:

\[ \frac{1}{\lambda} = \alpha \cdot \lambda^2 + g \cdot \lambda + d . \]  

This approximation resulted in negligible deviations (Figure 3). By using the relations (16), (19) and (20), it is obtained that:

\[ \frac{D}{A_y} \cdot 1 = \frac{2 \cdot \gamma_x}{\beta_x \cdot h \cdot A} \cdot (M_{sy} + c \cdot A) \cdot f(b) \cdot b^2 . \]  

#### 3.2.2. The criterion of local buckling of plates

Testing of the box girder stability was carried out in accordance with the standards group [13]. According to this standard, it is necessary to check the stability of the flange plate with the width \( b_1 \) and the thickness \( t_1 \) (Figure 4), the stability of the web plate above the longitudinal stiffener (length \( a \), height \( h_1 \) and thickness \( t_1 \) – Figure 4) as well as the stability of the web plate under the longitudinal stiffener (length \( a \), height \( h_2 \) and thickness \( t_2 \) – Figure 4).
Fig. 4. Elements of the box profile relevant for testing of the local stability

Testing of the stability of the flange plate segment (Figure 4) subjected to the action of normal compressive stress in the $x$ direction was carried out. This criterion is fulfilled if the following condition is satisfied:

$$g_2 = \frac{M_{ch} + c \cdot A}{\alpha_y \cdot h \cdot A} \cdot f \cdot M_{mb} + k_y \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \sigma_{x2} \leq 0,$$  \hspace{1cm} (25)

where:

$$\sigma_{x2} = \sigma_{cr} \cdot f \cdot (\gamma_{ns} \cdot v_i) - \text{critical stress},$$

$$\gamma_{ns} = 1.1 - \text{the general resistance factor},$$

$$v_i = 1.5 - \text{the factored load coefficient for load case 1},$$

$$\kappa_y - \text{a reduction factor according to Equation:}$$

$$\kappa_y = c_y \cdot \left(1 - \frac{0.22}{\lambda_y^2}\right) \leq 1 \text{ for } \lambda_y > 0.673,$$  \hspace{1cm} (26)

$$\kappa_y = 1 \text{ for } \lambda_y \leq 0.673,$$

where:

$$\lambda_y = \frac{f_{ch}}{K\sigma \cdot \sigma},$$  \hspace{1cm} (27)

$$c_y = 1.25 - 0.12 \cdot \psi \cdot \sigma, \quad c_s \leq 1.25,$$  \hspace{1cm} (28)

$$\psi \sigma - \text{the edge stress ratio of the plate, relative to the maximum compressive stress},$$

$$K\sigma - \text{a buckling factor according to [13],}$$

$$\sigma_y - \text{a reference stress according to Equation:}$$

$$\sigma_y = \frac{2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot (t_i / h)^2.$$  \hspace{1cm} (29)

$K\sigma$ depends on the ratio between the plate sides $a_x = a / h$. Now, this relation is to be analyzed.

In the cross section $I$ the vertical diaphragms are placed at the distance of $2h$, so that this ratio takes the value:

$$\lambda_y = a / b = 2 - h \cdot (f \cdot b) > 1.$$

Also, it is necessary to analyze coefficient $\psi$. It takes following value:

$$\psi = \frac{c_y \cdot \alpha_y - f \cdot k}{c_y \cdot \alpha_y}.$$  \hspace{1cm} (30)

where:

$$v_i = 1.5 - \text{the factored load coefficient for load case 1},$$

$$\sigma_1, \sigma_2 - \text{the stresses due to the factored load.}$$

For average values, this ratio can also be approximately written by the expression $\psi \sigma \approx 0.83 - 0.06 \cdot k$. By using this approximation, with appropriate transformation, it can be obtained:

$$K\sigma_y = \frac{8.2}{1.88 - 0.06 \cdot k}.$$  \hspace{1cm} (31)

$$\psi = \frac{K\sigma_y \cdot s \cdot f}{\sqrt{K\sigma_y}} - \text{constant.}$$  \hspace{1cm} (27.1)

In Figure 5 it is shown that the factor $\kappa_y$ takes the value 1 for the defined range of the ratio $k$, classification class 2m/M5 and girder material S235JRG2.

In order to apply the method of Lagrange multipliers, it is necessary to find the corresponding partial derivatives (5), in accordance with the expressions (7) and (25):

$$\frac{\partial g_1}{\partial k} = \frac{\partial g_2}{\partial k} = 0.$$

After some transformation, it is obtained:

$$\kappa_y = \left[\frac{f \cdot \alpha_y}{c_y \cdot k_y}\right].$$  \hspace{1cm} (32)

Fig. 5. Change of the coefficients $\lambda_{sp}$ and $\kappa_y$ as the function of the parameter $k$

From this expression, we can get the optimum value of the parameter $k$ according to the criterion of lateral stability of the flange plate. Using the obtained dependencies from the constraint function according to the criterion of stability of the flange plate, the constraint function can be written in the following form:

$$A_y(h) \geq \frac{M_{ch} / \alpha_y + f \cdot M_{mb} / \alpha_y \cdot k_{pl}}{\sigma_{x2} \cdot h - c / \alpha_y - f \cdot k_{c} / \alpha_y \cdot k_{pl}}.$$  \hspace{1cm} (33)

Testing of the local stability of the web plate in area 1 and area 2, whose dimensions are given in Fig. 5, was carried out. Besides the normal stresses in $x$ direction, there is normal stress in $y$ direction too, due to the action of wheel pressure.
Further on, load in y direction is analyzed: 

$$F_y -$$ the highest wheel pressure,

$$l_{ir} = 12.15 + 1.4 \cdot e$$ - the effective distribution length [13],

$$\kappa_{ir}$$ - a reduction factor for area 1:

$$\kappa_{iy} = 1.13 \cdot \left( \frac{1}{\lambda_{iy}} - \frac{0.22}{\lambda_{iy}^2} \right) \leq 1$$ for $$\lambda_{iy} > 0.831,$$ (39)

$$\kappa_{iy} = 1$$ for $$\lambda_{iy} \leq 0.831,$$

where:

$$\lambda_{iy} -$$ the non-dimensional plate slenderness for area 1 according to Equation:

$$\lambda_{iy} = \frac{f_y}{K_{\sigma_{iy}} \cdot \sigma_{iy} \cdot a / c_{ir}} = \frac{Ko}{\sqrt{K_{\sigma_{iy}} \cdot a / c_{ir}} \cdot (s / 4)},$$ (40)

$$K_{\sigma_{iy}} = 0.5 - \alpha$$ buckling factor for area 1 according to [13],

$$c_{ir} -$$ the width over which the transverse load is distributed (corresponds to $$l_{ir}$$).

Fig. 7 shows that for average and expected parameters values, the value of coefficient is $$\kappa_{iy} = 1$$, wherein classification class is 2 and material is S235JRG2.

On the basis of the obtained values the relation (34.1) can be written:

$$\sqrt{\left[ \frac{\sigma_{Sl1,i}}{\sigma_{Sl1,i}} \right]^2 + \left[ \frac{\sigma_{Sl1,y}}{\sigma_{Sl1,y}} \right]^2} - \left[ \sigma_{Sl1,i} \cdot \sigma_{Sl1,y} \right] \leq f_y / \gamma_m.$$ (34.1.1)

To maintain this relation the following condition must be fulfilled:

$$\sigma_{Sl1,i} \geq \sigma_{Sl1,y},$$ or the left side must be greater then the right side, whereby this dependences are given in relation to bridge crane span.

Similar procedure is applied for area 2.

This criterion is satisfied if the local stability condition is fulfilled:

$$\left( \frac{\sigma_{Sl2,i}}{f_{Sl2,i}} \right)^n - (\kappa_{iy} \cdot \kappa_{ix}) \left( \frac{\sigma_{Sl2,i} \cdot \sigma_{Sl2,y}}{f_{Sl2,i} \cdot f_{Sl2,y}} \right) \leq 1,$$ (34.2)

where:

$$\sigma_{Sl2,i} = \frac{M_{ir} + c \cdot A}{2 \cdot \alpha_i \cdot h \cdot A} + f \cdot \frac{M_{ir} + k_i \cdot c \cdot A}{\alpha_i \cdot b \cdot A} -$$ the maximum compressive stress in point 2 in x direction,
\[ \sigma_{sd,2} = \frac{\gamma \cdot F_y}{l_2} \] - the maximum compressive stress in point 2 in y direction, 
\[ f_{y,SD,2} = \kappa_{sd,2} \cdot f_y / \gamma_m \] - the critical compressive stresses, 
\[ e_{\gamma} = 1 + \kappa_{\gamma} \] - coefficients, 
\[ \kappa_{\gamma}, \kappa_{sd,2} - \text{a reduction factor for area } 2. \]

The same analogy exists for \( \lambda_{2y} \), as well as for \( \psi_{2y} \) and \( K_{sd,2} \). Analysis for y direction will be done later on.

\( c_{\gamma} = 1.25 \).

A reference stress \( \sigma_{2e} \) is obtained according to equation:
\[ \sigma_{2e} = \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \left( \frac{4}{3 \cdot s} \right)^2, \quad (35.2) \]

Since there is:
\[ \psi_{2y} = -\left(0.6 + 0.01 \cdot k\right), \quad (36.2) \]

it can be noted
\[ K_{2e,2} = 15.1 + 1.8 \cdot k + 0.0978 \cdot k^2, \quad (37.2) \]
\[ \lambda_{2y} = \frac{K_0}{K_{\sigma_{2y}}} \left(3 \cdot s / 4\right). \quad (38.2) \]

Fig. 8 shows that for average and expected parameters values, the value of coefficient \( \kappa_{2y} \) varies, wherein classification class is 2 and material is S235JRG2. Increasing the slenderness \( s \) leads to much lower values.

Further on, load in y direction is analyzed:
\[ l_y = 12.15 + 2 \cdot e \cdot h / s + h / 2 \] - the effective distribution length [13],
\[ \kappa_{2y} - \text{a reduction factor for area } 2. \]

There exists the same analogy for coefficient \( \lambda_{2y} \) as for \( K_{2y} \):
\[ \lambda_{2y} = \frac{f_y}{\sqrt{K_{\sigma_{2y}} \cdot \sigma_{\gamma}}} = \frac{K_0}{\sqrt{K_{\sigma_{2y}} \cdot a / \sigma_{\gamma}}} \cdot 3 \cdot s / 4, \quad (41) \]
\[ \kappa_{\gamma}, \kappa_{sd,2} = 1.2 \] - buckling coefficient [13],
\[ c_{\gamma} - \text{the width over which the transverse load is distributed (corresponds to } l_{2y}.\]

To do analysis, the ratio between the maximum compressive stress in this area and the compressive stress that occurred.
\[ \psi_{2y} = -\left(0.6 + 0.01 \cdot k\right), \quad (36.2) \]

Now (34.2.1) becomes as follows:
\[ \left(\frac{\sigma_{2e,2}}{f_{y,SD,2}}\right)^{1/e_2} + \left(\frac{\sigma_{2e,2}}{f_{y,SD,2}}\right)^{1/e_2} \leq 1, \quad (34.2.2) \]

Fig. 10 shows that for average parameters values and with load changing, this relation varies, where it was considered \( s = 160 \).
4. NUMERICAL REPRESENTATION OF THE RESULTS OBTAINED

Using the expression (23) the optimum value of the parameter \( k_1 \) according to the criterion of lateral stability is obtained. Optimum values of the parameter \( k_1 \) as a function of the coefficient \( e \), while \( k_a = 0.1 \), are presented in Table 1.

<table>
<thead>
<tr>
<th>( e )</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td>3.35</td>
<td>3.33</td>
<td>3.31</td>
<td>3.30</td>
</tr>
</tbody>
</table>

The expression (24) represents the objective function obtained from the constraint function according to the criterion of lateral stability and together with the objective function (7) it can be graphically represented. Fig. 11 shows how the position of the intersection point changes depending on the selection of material, where adopted values are \( L=20 \, m \) and \( Q=12.5 \, t \). Using the expression (32) the optimum value of the parameter \( k \) according to the criterion of lateral stability is obtained. Optimum values of the parameter \( k_2 \) as a function of the coefficient \( e \), while \( k_a = 0.1 \), are presented in Table 2.

<table>
<thead>
<tr>
<th>( e )</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_2 )</td>
<td>4.20</td>
<td>4.30</td>
<td>4.40</td>
<td>4.55</td>
</tr>
</tbody>
</table>

The expression (33) represents the objective function obtained from the constraint function according to the criterion of local stability and together with the objective function (7) it can be graphically represented. Fig. 12 shows how the position of the intersection point changes depending on the selection of material, where adopted values are \( L=20 \, m \) and \( Q=12.5 \, t \). In order to perform a comparative analysis, it is necessary to take into consideration the recommendations specified in the standard as well as those given by crane manufacturers [14]. Serbian crane manufacturers recommend that the minimum value of the width \( b_1 \) should be \( b_1 > 20 \, \text{cm} \), wherefrom it is obtained that:

\[
k \leq f \cdot h / 20 ,
\]

while stability condition of top flange plate, after proper transformation, which reads:

\[
k \geq \frac{s \cdot f}{65 \cdot e} \cdot \sqrt{\frac{23.5}{f_y}}.
\]

If the expressions (7) and (24) are made equal, the dependence of the parameter \( k_1 \) according to the criterion of lateral stability is obtained:

\[
k_1 = F \left( s, e, h, K_1, K_2, M_{cr}, M_{cr}, f_y, c, k_a \right) .
\]

If the expressions (7) and (33) are made equal, the dependence of the parameter \( k_2 \) according to the criterion of local stability is obtained:

\[
k_2 = F \left( s, e, f, h, Q, M_{cr}, M_{cr}, \alpha, \alpha, k_2, f_y, \nu, E \right) .
\]
Figures 13-14 present optimum geometric parameters’ values that are obtained for characteristic load capacities and spans of double-girder bridge cranes. This procedure enables quick and efficient determination of optimum value of parameter $k$, according to critical function.

5. CONCLUSION

The paper defined optimum dimensions of the box section of the main girder of the bridge crane in an analytical form, by using the method of Lagrange multipliers, according to lateral stability and local stability of plates criteria. [13]. The objective function is the minimum mass, i.e. the minimum area of the cross section, where the given constraints are satisfied: lateral and local stability, manufacturing technology and top (compressed) flange plate stability. It can be observed that greater heights of box girder cross section are obtained according to local stability criterion [13] (consequently greater cross section areas) in comparison with those which are calculated due to lateral stability criterion, which is taken from national standard. So, the local stability of plates is the primary criterion [13]. The results obtained may be of great use to the engineer-designer, particularly in the first phase of the design procedure when the basic dimensions of the main girder of the bridge crane, as its most responsible part, are defined. In addition, the usage of the method of Lagrange multipliers is justified because the optimization results are obtained in analytical form, which allows getting conclusions about influences of particular parameters and further researches toward mass reduction.

REFERENCES