MATHEMATICAL MODEL FOR INVESTIGATION OF FACE GEAR TOOTH SURFACE MANUFACTURED BY NEW CUTTING EDGES OF SPIROID HOB HAVING ARCHED PROFILE IN AXIAL SECTION

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Abstract: We have defined the equations of typical surfaces and edges of spiroid hob having arched profile in axial section. Knowing these equations mathematical model have been worked out for determination of face gear tooth surface points which are manufactured by the received, new cutting edges during the resharpening along the hob face surface. Using this model it is possible to keep the face gear profile points in the axial section of hob on the profile shape tolerance, hereby the resharpening limit angle of the hob could be determined.

Keywords: arched profile, spiroid, backward turning, cutting edge

1. INTRODUCTION

Knowing the main characteristics and production geometry of hob is important for the correct and appropriate quality production [3, 4, 5]. The elements of the face gear hob is equal with the elements of the worm co-operating as a gear drive with the face gear by direct motion mapping (Figure 1). The module and the thread pitch exactly match for the similar elements of the worm. The diameter vary because of the resharpening provision [3, 4, 5]. The basic worm, from which the hob is established, is suitable for the type of worm used in a worm gear drive.

2. THE GEOMETRIC ESTABLISHMENT OF SPIROID HOB

Knowing the equations of the cutting edge, the face surface and the backward turned side surfaces of hob and the logarithm spiral using our new method the hob resharpening area could be determined so that the face gear profile is situated inside of the profile error tolerance.

Fig.2. The geometric establishment of logarithm spiral

The described equation of logarithm spiral in polar coordinate is (Figure 2) [3, 4]:

\[ r_i = a_m \cdot e^{k \cdot \varnothing_i} \]  

where the \( a_m \) constant is:

\[ a_m = \frac{D_1}{2 \cdot e^{k \cdot \varnothing_i}}. \]

The dimension of reduction of the \( c_1 \) tooth height is (Figure 3):

\[ c_1 = \frac{D_1}{2} - a_m \cdot e^{k \cdot \varnothing_i}. \]

Fig.3. Backward turning parameters
2.1. The equations of cutting edge, face- and backward turned side surface in case of $\gamma_0 < 5^\circ$

Given the two parametric vector - scalar function of conical wrapping surface in case of low side [1]. The $\gamma_0$ helix angle is smaller than $5^\circ$ that is why the face surface is plane ($x^\prime = 0$) which is passed the hob axis [3, 4] (Figure 4). So the equation of the cutting edge is:

$$
\begin{align*}
&x^f = 0 \\
y^f = \eta \cdot \cos (\theta + \phi_r) + p_r \cdot \phi \cdot \cos \phi_r \\
z^f = p_a \cdot (\theta + \phi_r) + \sqrt{p_a^2 - (K_e - \eta)^2} + z_{avw} 
\end{align*}
$$

(4)

In case of the high side electing the appropriate indications the equations of the face surface, the backward turned surface and the cutting edge could be defined similar to the low side.

Knowing the equations of the cutting edge (4), the backward turned side surface (5), the face surface ($x^f = 0$) and the logarithm spiral of the hob a computer program has been carried out for the 3D modelling of hob tooth and the verification of the received equations. Using this computer program the determination of the hob tooth surface points (face surface, backward turned side surface head ribbon) and the CAD modelling of hob tooth could be determined.

2.2. The equations of cutting edge, face- and backward turned side surface in case of $\gamma_0 \geq 5^\circ$

Given the two parametric vector - scalar function of conical wrapping surface in case of low side [1]. The $\gamma_0$ helix angle is larger than $5^\circ$ that is why the face surface is a closed Archimedean helical surface (Figure 6) [3, 4].

Generation of this surface: a half-line perpendicular to the centre line of the helicoid is drawn in accordance with face thread parameter $p_h$ following a translational and $p_r$ radial motion parallel to its rotation, too (Figure 6) [3, 4]. On a right-hand thread-milling cutter, its face surface follows a left-hand thread:

$$
\vec{r}^H_g = \eta \cdot \vec{j}
$$

(6)
So the equations of face surface on the K_{1F} coordinate system are:

\[
\begin{align*}
  x_{1F}^H &= -\eta \cdot \sin(\varphi + \varphi_{ob}) - p_r \cdot \vartheta \cdot \sin \varphi \\
  y_{1F}^H &= \eta \cdot \cos(\varphi + \varphi_{ob}) + p_r \cdot \vartheta \cdot \cos \varphi \\
  z_{1F}^H &= -p_h \cdot (\vartheta + \varphi_{ob}) 
\end{align*}
\]

(7)

The cutting edge of hob is given by the intersection of the worm surface and the face surface:

\[
\begin{align*}
  x_1' &= -\eta' \cdot \sin \left[ \frac{p_r - (\vartheta + \varphi_1)}{p_a} \right] - \frac{1}{p_a} \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} - p_r \cdot \vartheta \cdot \sin \varphi_1 \\
  y_1' &= \eta' \cdot \cos \left[ \frac{p_r - (\vartheta + \varphi_1)}{p_a} \right] + \frac{1}{p_a} \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} + p_r \cdot \vartheta \cdot \cos \varphi_1 \\
  z_1' &= p_a \cdot (\vartheta + \varphi_1) + \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} + z_{aw}
\end{align*}
\]

(8)

The equation of the backward turned surface is the following without derivation:

\[
\begin{align*}
  x_{aw} &= -\eta' \cdot \sin \left[ \frac{p_r - (\vartheta + \varphi_1)}{p_a} \right] - \frac{1}{p_a} \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} - \eta \cdot \vartheta \cdot \sin \varphi_1 \\
  y_{aw} &= \eta' \cdot \cos \left[ \frac{p_r - (\vartheta + \varphi_1)}{p_a} \right] + \frac{1}{p_a} \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} + \eta \cdot \vartheta \cdot \cos \varphi_1 \\
  z_{aw} &= p_a' \cdot (\vartheta + \varphi_1) + \sqrt{p_{a'}^2 - \left( K_s - \eta \right)^2} + z_{aw}
\end{align*}
\]

(9)

In case of the high side selecting the appropriate indications the equations of the face surface, the backward turned surface and the cutting edge could be defined similar to the low side.

3. MATHEMATICAL MODEL FOR DETERMINATION OF THE FACE GEAR TOOTH SURFACE POINTS

Knowing the two parametric vector – scalar function \( r_{1F}(\eta', \varphi_1) \) which is generated by the cutting edge of hob the purpose is to generate the face gear tooth surface points during resharpening along the hob face surface. During resharpening the received face gear tooth surface by wrapping are searched with the common solution of the wrapping worm and the Connection I. statement in the K_{2F} (x_{2F}, y_{2F}, z_{2F}) coordinate system.

Solving this problem we have worked out a new mathematical model (Figure 9).

Based on Figure 7 the position vector coordinates of any point on discreet resharpening angle owing to profile generator:

\[
\begin{align*}
  P_r \left[ 0, \eta', p_a \cdot \vartheta + \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} + z_{aw} \right] \\
  P_h \left[ 0, \eta', p_a \cdot \vartheta - \sqrt{p_{a}^2 - \left( K_s - \eta \right)^2} + z_{ah} \right]
\end{align*}
\]

(10) (11)

The contact curve \( \varphi \) which is in coordinate system K_{aw} (\xi, \eta, \zeta) along the axis z_{1F}, has axial parameter p_a and along the axis y_{1F}, it has radial parameter p_r and it gives helicoidal movement, the contact curve touches a conical surface in coordinate system K_{aw} (\xi, \eta, \zeta), which before the rotating movement overlaps the coordinate system K_{aw} (Figure 8) [1, 3, 6].
The transformation matrix between the \( K_{1F} (x_{1F}, y_{1F}, z_{1F}) \) hob rotational and the \( K_{2F} (x_{2F}, y_{2F}, z_{2F}) \) face gear rotational coordinate systems is:

\[
M_{2F,1F} = M_{2,1r} \cdot M_{1r,1F} = \\
\begin{bmatrix}
- \cos \phi_1 \cdot \cos \phi_2 & \cos \phi_1 \cdot \sin \phi_1 & - a \cdot \cos \phi_1 + b \cdot \sin \phi_2 \\
\sin \phi_1 \cdot \cos \phi_2 & - \sin \phi_1 \cdot \sin \phi_1 & \cos \phi_1 \cdot \cos \phi_2 \\
\sin \phi_1 & \cos \phi_1 & 0 \cdot c
\end{bmatrix}
\]

(14)

The relative velocity \( \tilde{v}^{(12)}_{1F} \) between the two surfaces can be defined based on the transformation between the \( K_{1F} \) worm and the \( K_{2F} \) worm gear coordinate systems in the \( K_{2F} \) system:

\[
\tilde{v}^{(12)}_{1F} = M_{1F,2F} \cdot \frac{dM_{2F,1F}}{dt} \tilde{r}_{1F}
\]

(15)

where:

\[
P_{ik} = M_{1F,2F} \cdot \frac{d}{dt}(M_{2F,1F})
\]

(16)

the kinematic generation matrix:

\[
P = \\
\begin{bmatrix}
0 & -1 & - l_{i1} \cdot \cos \phi_1 & - b \cdot l_{i2} \cdot \cos \phi_1 \\
1 & 0 & l_{i1} \cdot \sin \phi_1 & b \cdot l_{i2} \cdot \sin \phi_1 \\
l_{i1} \cdot \cos \phi_1 & - l_{i1} \cdot \sin \phi_1 & 0 & a \cdot l_{i2}
\end{bmatrix}
\]

(17)

Defining the normal vector in the \( K_{1F} \) coordinate system is:

\[
\tilde{n}_{1F} = \frac{\partial \tilde{r}_{1F}}{\partial \eta} \times \frac{\partial \tilde{r}_{1F}}{\partial \phi}
\]

(18)

On the connecting tooth surfaces of elements, as on wrapping each other surfaces the contact curve can be defined by the concomitant solving of expression of Connection I. statement

\[
\tilde{n}_{1F} \cdot \tilde{v}^{(12)}_{1F} = \tilde{n}_{2F} \cdot \tilde{v}^{(12)}_{2F} = \tilde{n} \cdot \tilde{v}^{(12)} = 0
\]

(19)

contact equation and describing tooth surfaces vector – scalar function [3].

Based on these the common solution of the two parametric vector - scalar function of wrapping surface and the connection equation gives the face gear tooth surface points in the \( K_{2F} \) face gear rotation coordinate system:

\[
\begin{bmatrix}
\tilde{r}^{p}_{1F} \\
\tilde{r}^{p}_{1F} \cdot \tilde{v}^{(12)}_{1F} = 0 \\
\tilde{r}^{p}_{2F} = M_{2F,1F} \cdot \tilde{r}^{p}_{1F}
\end{bmatrix}
\]

(20)

4. CONCLUSION

We have defined the equations of face surface, backward turned side surface and cutting edges of Spiroid hob having arched profile in axial section [2]. Knowing these equations mathematical model have been worked out for determination of face gear tooth surface points which is manufactured by the received, new cutting edges during the resharpening along the hob face surface.

Using this model the production geometric analysis of face gear tooth surface points in the hob axial section which is manufactured by new cutting edges and keeping of the face gear tooth profile in the function of the hob resharpening angel in the profile error tolerance are possible.

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