

ANALYSIS OF PROPULSION UNIT MATHEMATICAL MODEL

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Abstract: The paper deals with drive unit mathematical model behavior in function of design parameters change for different electric driven vehicle drive line design selection. The mathematical model for energetic optimization of drive unit into drive line has been created, which helps to determine the efficiency map of transmission. A parameter vector has been formed by the mathematical model, which contains the independent variants of drive unit. These variants have different effect on the model behavior. In this paper two independent variants have been examined, like cog width and helix angle.

Key words: *efficiency, design parameters, independent variables, vehicle*

1. INTRODUCTION

Different power-level electric and hybrid vehicles have been developed at the Széchenyi István University for more years. One of the main research directions is electric vehicle drive line modeling and examination. Today one of the key problems is energy storing, thus short travelling distance, which makes energetic examination of electric vehicles highly necessary. One of the main elements of energetic examination is the drive line, which includes the PMS motor and mostly the inhering drive unit. In case of a given vehicle the necessity and transmission of drive can be determined by complex examination. The demand of installing drive unit arises the question of the most appropriate construction and selection. The main difficulty in design is that drive unit efficiency is considered to be constant in the whole operation interval, while it shows significant deviation in different operational intervals. That is why the drive unit mathematical model is needed, by which an energetically optimized drive can be fit into vehicle drive line. The mathematical model includes determination of independent drive unit design parameters. The main question is their effects on the model therefore examinations must be carried out. Among independent variants two parameters have been determined, which have negligible effect on model according to the examinations. In this paper these parameters' common effects on the model are introduced.

2. THE MATHEMATICAL MODEL FORMULATION

Two ways have been chosen for creating mathematical model. In the first case drive unit dynamical model was created, which describes the given drive unit not at a given work point only, but at transient states during acceleration and deceleration processes. The advantage of this modeling procedure is the description of entire drive unit [1].

Second option is creating model based on analytical way applying formulas [1]. In this case the created model does not handle transient, only work point states, but claims less computing and computation time.

By both models a given drive unit can be optimized using the following procedure. In case of a selected or created test cycle it is segmented into work points (torque, RPM). Probability of vehicle position at a certain work point is assigned to the work point, thus drive unit energy loss can be determined. By the cumulated energy loss each drive unit variants can be compared and evaluated [2].

3. DRIVE UNIT PARAMETER VECTOR DETERMINATION

Drive unit model is constructed so, that drive unit loss is determined by steady-state examination at work points. This work point examination makes both dynamical and analytical mathematical model application possible during

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optimization. Parameter determination has key importance in model construction, by which drive unit can be optimized [3]. These parameters are such independent variables, which can unambiguously determine a given drive unit [4]. Parameters create a vector space, where drive unit optimization is realized. Drive unit parameter vector:

$$Pg[i, m, a_w, b, \alpha, \beta] \quad (1)$$

transmission ratio: i ,
 module: m ,
 center distance: a_w ,
 gear width: b ,
 pressure angle: α ,
 helix angle: β ,

4. MATHEMATICAL MODEL EXAMINATION

During the examination of efficiency field by the model it was stated that drive unit efficiency in the whole load interval depends minimal on speed, but load torque affects it significantly [1]. Realizing this model parameter dependency is only examined as a function of load torque. Thus a drive unit type has been chosen, of which parameter vector has also been determined:

$$P_h[1; 1,5; 78,75; 15; 20; 0] \quad (2)$$

Then one of the parameters left as independent variant, while the rest of them is set and at the same RPM ($n=500$ 1/min) in the given load interval ($M=0-60$ Nm) is the model examined in order to determine the selected drive unit variant sensitivity.

5. HELIX ANGLE EFFECT ON THE MATHEMATICAL MODEL

Concerning publications it was realized that energetic examination and modeling of helical gears connection are hardly introduced. Foreign papers showed some information about examination results, e.g. Haizuka et al. They made experimental research on helix angle effect on friction power loss in case of helical gears. The result was that increasing helix angle caused increasing power loss.

Own designed drive unit was examined, therefore drive unit parameter vector was determined:

$$P_h[1; 1,5; 78,75; 15; 20; f(\beta)] \quad (3)$$

$$f(\beta) = 0 \dots 35, [^\circ] \quad (4)$$

It can be easily seen that helix angle was changed $0 \dots 35^\circ$, since in practice it is typically applied to 30° . Further angle increasing may cause technology problems and the axial bearing loads can significantly increase.

Regarding cog angularity examination it was realized that the model efficiency increased till around $23,55^\circ$ helix angle. However, this increasing was only a few ten per thousand order of magnitude, which is mainly constant and after this efficiency starts to decrease (Figure 1 and Figure 2). Decreasing was also only a few ten per thousand order of magnitude in the examined interval. Concerning the above it can be concluded that cog angularity has negligible effect on energetic optimization.

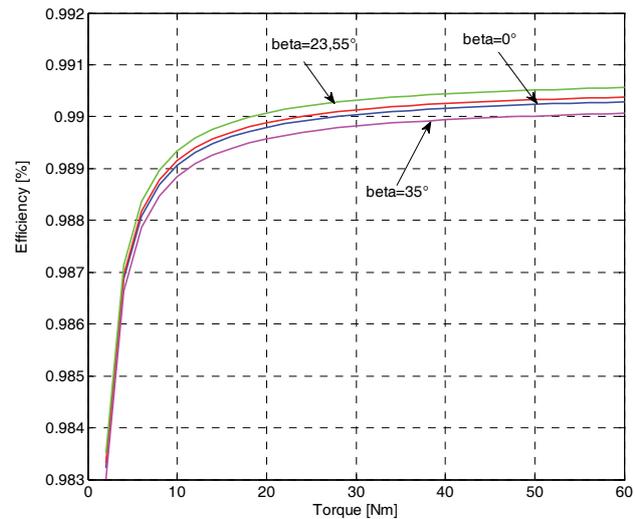


Fig.1. Cog angularity change effect on drive unit efficiency in a given load interval (examination interval: $n=500$ [RPM], $M=0-60$ [Nm])

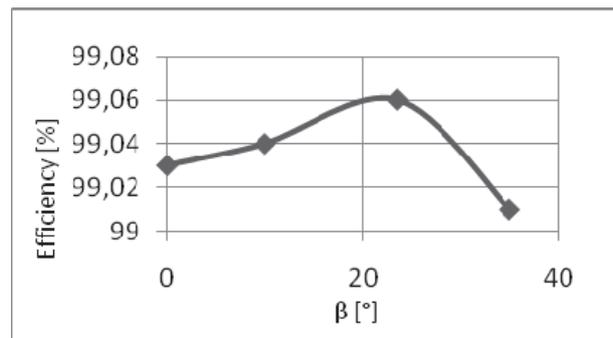


Fig. 2. Cog angularity effect on drive unit efficiency

6. GEAR WIDTH EFFECT ON MATHEMATICAL MODEL

First step is to determine drive unit parameter vector:

$$P_h [1; 1,5; 78,75; 15; 20; f(\beta)] \quad (5)$$

$$f(b) = 10 \dots 40, \quad (6)$$

Gear width is changed from 10 to 40 during examination. It can also be stated that gear width change has minimal effect on model efficiency (Figure 3).

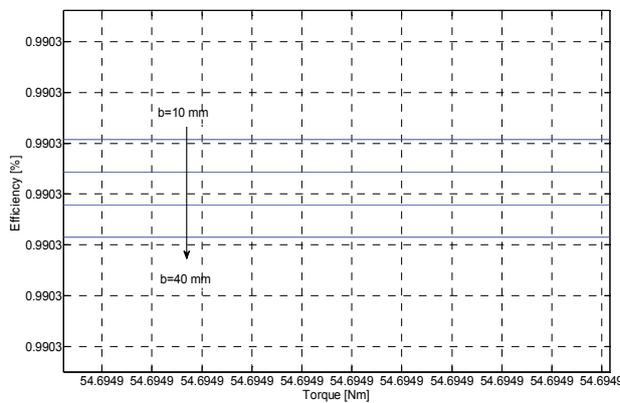


Fig.3. Gear width change effect on drive unit efficiency in a given load interval
(examination interval: $n=500$ [RPM], $M=0-60$ [Nm])

Mathematical model analysis showed that from loss sources three are affected by gear width:

- sliding loss (specific line load),
- oil churning loss,
- air windage loss.

In selection of models describing different losses the main intention was to design a reality like model, while not being very complex applying only a few factors, therefore the following models were used to handle the introduced losses:

a) Sliding friction modeling with Schlenk relation [6]:

$$\mu_z = 0,048 \cdot \left(\frac{F}{b \cdot v_{\Sigma C} \cdot \rho_{redc}} \right)^{0,2} \cdot \eta^{-0,05} \cdot R_a^{0,25} \cdot X_L \quad (7)$$

b) Churning oil loss described by Niemann approximating relation [7]:

$$P_0 = \frac{b \cdot b_m \cdot v^{\frac{3}{2}}}{2,72 \cdot 10^6} \quad (8)$$

c) Air windage loss is determined by relation applied in Loewenthal disc friction examination [8]

$$P_{l1} = 1,16 \cdot 10^{-8} \cdot \left(1 + 4,6 \cdot \frac{b}{d_1} \right) \cdot n^{2,8} \cdot d_1^{4,6} \cdot (0,028 \cdot \mu + 0,019)^{0,2} \quad (9)$$

$$P_{l2} = 1,16 \cdot 10^{-8} \cdot \left(1 + 4,6 \cdot \frac{b}{d_2} \right) \cdot \left(\frac{n}{i} \right)^{2,8} \cdot d_2^{4,6} \cdot (0,028 \mu + 0,019)^{0,2} \quad (10)$$

Notation:

- μ_z : sliding friction coefficient [-],
- F : tangential force at the base circle [N],
- b : gear width [mm],
- R_a : arithmetic mean roughness [\square m],
- $v_{\Sigma C}$: sum speed at operating pitch circle [m/s],
- X_L : lubricant coefficient [-],
- η : lubricant dynamic viscosity at working temperature [Pas],
- ρ_{redc} : reduced radius of curvature at pitch point [mm],
- n : gear absolute speed [RPM],
- b_m : immersion depth [m]; [mm],
- d : reference diameter [mm],
- μ : dynamic viscosity [Ns/m²],

Relations describing loss sources showed that sliding friction loss is decreased by increasing gear width, since specific line load on joint cogwheels decreases, while oil churning and air windage loss increasing proportionally.

In the examined load interval ($n=500$ [1/min], $M=0-60$ [Nm]) valid for a working phase of low-power city electric car, it can be definitely stated that the helix angle and gear width effects are negligible on the model, since their effect on drive unit is few per thousand order of magnitude in the examined interval.

5. CONCLUSION

Applying the introduced model examination design variants can be decreased, because gear width and helix angle can be left out from the parameter vector. Therefore the optimization algorithm will be much more simple and faster so, that its accuracy will not worsen significantly.

REFERENCES

- [1] József Polák, István Lakatos, Efficiency optimization of electric permanent magnet motor driven vehicle, The Journal of the Faculty of Technical Sciences, Machine Design, Vol. 7, Novi Sad, 2015,
- [2] L. GUZZELLA, A. SCIARRETTA: Vehicle Propulsion Systems, Springer-Verlag, Berlin Heidelberg, 2005, ISBN 978-3-540-25195-8
- [3] Polák József, Vida Bálint, Hajtómű részterhelésének veszteségvizsgálata és annak jelentősége, IFFK 2013, Budapest, 2013, augusztus 28-30.
- [4] Dr.Zsáry Árpád, Gépelemek II. Budapest, 1990, Nemzeti Tankönyvkiadó Rt. ISBN 963 19 1166 7
- [5] Haizuka, S., Naruse, C., Yamanaka, T., "Study of Influence of Helix Angle on Friction Characteristics of Helical Gears", Tribology Transactions, Vol. 42, No. 3, pp. 570-580, 1999.
- [6] Dirk Strasser: Einfluss des Zahnflanken- und Zahnkopfspeles auf die Leerlaufverlustleistung von Zahnradgetrieben, Dissertation zur Erlangung des Grades Doktor-Ingenieur, Fakultät für Maschinenbau, Ruhr-Universität Bochum, 2005
- [7] Erney György: Fogaskerekek, Műszaki Könyvkiadó, Budapest, 1983
- [8] John J. Coy, Dennis P. Townsend, Erwin V. Zaretsky: Gearing, NASA Reference Publication 1152, AVSCOM Technical Report 84-C-1 5, 1985