OPTIMAL DESIGN FUNDAMENTALS OF FATIGUE LOADED MACHINE SPRINGS

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Abstract: The aim of this paper is to present results of using fundamental machine element design principles into re-designing optimally heavy duty springs used in terrain machinery and in industry. Use of standard procedures often results in recurring fatigue fracture failures. This reveals need for correcting also the present standards. Analytical calculations reveal as main causes of failures the local bending due to eccentric highly impact force application at squared and ground ends and wearing away of the shot peening protection. Optimum design is used to solve the problem by finding the optimal spring. Goals are minimisation of wire volume, space restriction, desired spring rate, avoidance of surging frequency and achieving reliably long fatigue life. Available fatigue dimensioning methods are used with amplitude-mean stress diagrams and S-N curve approaches. Conclusions are verified by using full 3D solid FEM analysis by which the stresses and also strains, deformations and natural frequencies and modes are obtained. Then FEM is used to optimally fine tune and validate the best result.

Key words: Industrial optimisation, helical springs

1. INTRODUCTION

Background for this study is observation that conventionally designed helical springs did not have the expected long fatigue life time promised by standards. Analyses of many case studies have given clue that some overlooked effect may have contributed strongly. Among these are highlighted the highly impacting bending and torsional stress peaks due to non symmetric pressure application at ground spring ends. The conventional standards of dimensioning do not take these effects into account. This case study highlights the typical state of affairs in machine design area when design is based on safety factors or mean strength divided by mean stress. Conventional approach neglects scatter in stress calculation procedures and also in strength estimates. Standard fatigue life estimates are based on static strengths and existence of only torsional stresses and give widely differing answers. This scatter can be taken into account using probabilistic and fuzzy approach. Generally optimisation is not yet often applied to practical problem according to Hernandez and Fontan. One obstacle is difficulty of goal formulation and understanding of basic principles of machine design. Another obstacle is that problems are highly non-linear and involve mostly discrete variables. Gradient type methods have proved inapplicable without local smoothing. Several methods exist which work can be applied without pre-smoothing. One easy to use and robust algorithm is to generate a restricted number of virtual prototypes and then select the most appealing one. The goals in this study are the following. First the main mechanisms causing failure are identified and corrective redesign ideas are generated. Then systematic optimisation approach is activated. It includes non preferentially models for dimensioning and fatigue life estimation presented in texts of Norton, Shigley and Mischke and Spotts et al. The goals are formulated to maximise fuzzy satisfaction on performance of deflection vs. load behaviours, reliable fatigue life, dynamic behaviour and space constraints.

2. FUNDAMENTAL DESIGN PRINCIPLES

State of the art survey of engineering design principles reveals many principles are accepted provisionally as pragmatic guidelines and not final truths. Engineering work efficiency needed Successful results are based on the following basics: First individual competence of engineers is needed. Then group competence and synergy by a good teamwork are needed. It is recognized that over 80% of the problems in engineering work are due to misunderstanding between colleagues and the rest is due to lack of know how or misunderstanding of good design practices.

2.1 Design Definitions

There is some consensus that any machine element combines three aspects:
Function F. This aspect is invisible but real item which tells what the device should do to be desired.
Geometry G. This aspect means appearance, form and dimensions, like CAD drawing.
Materials M. This aspect includes materials selection. Any physical machine is union of all these.

Machine = F and G and M

At first Machine is a concept or virtual machine.

Design tools are applied to this concept to realize it. Design variables are defined to be obtained in design but parameters are accepted as given. Design requirement definitions. These tools are needed to define expected performance and environment.
Evaluation criteria definition. These are statements defining desirable characteristics. These assist designer to define the optimum.

2.2 Axiomatic Design Tool.

Suh [5] proposes two axioms as design concept tools.

A. Minimization of information axiom

Relevant information is disturbed by too much irrelevant information.

B. Independence of functions axiom. Complex unknown interactions between functions are not good.

2.3 Concept Tools for Optimum Design Work

Classical optimisation. Using this approach three separate items are addressed in a task.

Maximise (Minimise) objective function K subject on inequality R > 0 and equality E = 0 constraints.

Fuzzy design. Difference between K, E and R is often fuzzy. This problem due to fuzziness can be clarified by using consciously “fuzzy design”.

Goals and constraints combined fuzzily. These are combined to the goal of maximization of probable net profit

\[ \text{Netto profit} = P \cdot F \cdot A \cdot (\text{Brutto} - \text{Expenses}) \]  

(1)

Profit reducing factors \( P \) may be chosen in two ways:

- \( P_i = \text{Pr}(\text{xx}(\text{Ig})) \) is probability of achieving property no. \( i = IG \), called \( xx(IG) \) = (cost etc), \( P_\text{r}(\text{research}) \), \( P_\text{d}(\text{development}) \), \( P_\text{p}(\text{production}) \), \( P_\text{m}(\text{Marketing}) \).

- \( P_i \) are fuzzy satisfaction functions \( P_i = P(G_i) \) = satisfaction on event \( G_i \).

2.4 An Example to Clarify the Principles

A typical goal is to transmit maximal force \( F \) from one part of construction to another through a minimal load bearing area \( A \). It may be a tensilely loaded machine element like a cylindrical rod.

2.4.1 Constraints as goals

Goal is \( Q = F/A \) is also stress which is subjected to endurance constraint: \( Q = \text{stress} = \sigma \). Then margin of safety \( Z = R - \sigma > 0 \), here \( R \) = strength, e.g. UTS.

2.4.2 Goal formulations

The best goal formulation should give best optimum.

A: Product of properties is to be maximized

Now design variables \( F \) and \( A \) are chosen as properties

\[ Q = \frac{F}{A} = \frac{1}{A} = f_1 Q_1 \cdot f_2 Q_2 = Q_1 \cdot Q_2 \]  

(4)

Goal formulation as product of properties is not reasonable and acceptable by an end user.

B: Weighted sum of properties to maximum

\[ Q \Rightarrow f_1 Q_1 + f_2 Q_2 = f_1 \frac{F}{F_0} + f_2 \frac{A}{A_0} \cdot f_1 + f_2 = 1 \]  

(5)

Problems are how to formulate weights reasonably and to choose scaling variables to obtain a dimensionless goal

C: Satisfaction on event \( Q \) to maximum

New design variables are properties which considered as events. The design goal is formed using as evaluation criteria simple fuzzy functions.

\[ G = \text{Event}(F) \text{ and Event}(A) = G(F) \text{ and } G(A) \]

\[ P(G) = P(G_F) \cdot P(G_A) \]  

(6)

\[ \text{Netto profit} = P(G) \cdot \text{Brutto profit} \]

This goal satisfies the Axiomatic design principles

Axiom 1: Information is minimal by only two properties

Axiom 2: Events \( F \) and \( A \) are treated as independent

Fig. 1. Two goal formulations for maximizing force and minimizing cross sectional area \( A \) in an example

2.4.3 Comparison of goal formulations

Satisfaction approach C. Main advantages are:

a) It expresses what the customer wants

b) All design requirements are reasonably fulfilled.

c) Risky design and overdesign are avoided.

d) The best and next best answers are obtained unambiguously on a numerical scale 0 to 1 with no need for further additional interpretations.

e) Goals may also be improved when needed. It may happen that some desired ranges were not realistic.

Conventional formulations A and B.

Main advantage is simple formulation. Serious disadvantages are that they do not express well what the customer wants. These goals recommend property ranges which are unacceptable to customer.
3. MATERIALS AND METHODS

3.1. Object of Study

The object of study is a range of helical compression springs which are used in heavy duty application with very high life reliability requirements. Their main function is to store energy from displacements and also withstand shocks and impacts. Ground ends cause local bending moment maximum as illustrated in Fig.2.

Fig.2. Definitions of a helical spring with ground ends

Definitions of spring variables and fuzzy satisfaction functions are shown in Fig.3. The fuzzy function \( p_x \) has max height unity, but area is not unity. Probability density function of mean value of property variable \( xx \) is \( \text{pdf}(xx) \), height is not unity but area is unity.

Fig.3. Definitions. a) Definition of spring variables. b) Definitions of a satisfaction function

3.2. Design Goal Formulation

3.2.1. The overall design goal

This is to maximise the satisfaction \( P(G) \) of end user customer on the realised design event \( G \). It is a union of partial design events. The “cost event” means now volume of the spring wire. Reliable functioning of the spring is achieved by using as desired properties the respective safety factors, SF. The space allotted for the spring is restricted in width and height.

\[
G = G_1(x(1)=\text{cost}) \cdot G_2(x(2)=N_{\text{good}}) \cdot G_3(x(3)=N_{\text{safe}}) \\
\cdot G_4(x(4)=N_{\text{spont}}) \cdot G_5(x(5)=N_{\text{aux}}) \cdot G_6(x(6)=N_{\text{life}}) \\
\cdot G_7(x(7)=f_{\text{surge}}) \cdot G_8(x(8)=k_{\text{spring}}) \cdot G_9(x(9)=\text{height})
\]  

Total satisfaction of this design event is product of partial functions

\[
P(G) = P(G_1) \cdot P(G_2) \cdot P(G_3) \cdot P(G_4) \cdot P(G_5) \cdot P(G_6)
\]

3.2.2. Design satisfaction functions

These are defined on four points to give a trapezoidal form, Fig.3. Value of property, \( xx(IG) \) with index IG is on horizontal scale and satisfaction \( p_x \) on \( xx \) is on vertical scale, ranging from 0 no good to 1 unit or fully good. Machine element design methods are based on strength diagrams and theoretical and empirical relationships between their variables. If the mean value of a stochastic property \( xx(IG) \) is within the most satisfactory range, and if its design value is normally distributed, then worse or better values may occur. Design is robust axiomatically when fuzzy range can be produced well within the design range.

3.2.3. Discrete variables

These are diameter of wire \( d \) (Id), diameter of spring helix \( D \) (Idd) and total number of coils \( N \) (INtot). This selection includes only the reasonable options. Optimum strategy was done by exhaustive search loops. Preliminary choices are a) Choice of assuming not \( (Isp=1) \) shot peening or yes \( (Isp=2) \), b) Choice of impact factor \( V \), c) choice of an assuming of volume fraction of inclusions.

1. Loop for material selection \( Im = 1 \) to \( N_{\text{mat}} \),
2. Loop for helix diameter \( D \) (Idd) variation,
3. Loop for wire diameter \( d \) (Id) variation
4. Loop for total number of coils \( N \) (INtot) variation.

Inside the loops the best optimum choice is made.

3.2.4. Material property data for optimisation

Now a reasonable selection for materials is ASTM A232 chrome Vanadium steels AISI 6150. According to Norton [2] it is suitable for fatigue loading. It is good for shock and impact loads for temperatures to 220°C. Maximum wire diameter recommendation is 12mm. Shear modulus is \( G = 79000 \) MPa. Ultimate tensile strength depends on the diameter \( d \) of the wire

\[
R_m = S_{\text{ult}} = \frac{A}{d_m}, \quad R_m[MPa],
\]

\[
d[mn], \quad A = 1880[MPa]m, \quad m = 0.192
\]

The following definitions of strength values are derived by empirical relationships from the static tensile strength. Here \( R_e \) is yield strength in tension, \( S_{\text{ult}} \) is yield strength in torsion and \( S_{\text{ult}} \) is ultimate strength in shear
3.2.5 Load stresses

Load to the spring comes from a cam mechanism. Nominal shear stress depends on load force.

\[ \tau_n = T_{nf} \cdot F = \frac{8 D}{\pi d^3} \cdot F, \]
\[ T_{nf} = \frac{8 D}{\pi d^3} \]

Shear stress is maximal in the inner coil due to smallest curvature.

\[ \tau_{xy,F} = K_w \cdot \tau_n = K_w T_{nf} \cdot F = T_F \cdot F \]
\[ T_F = K_w T_{nf} \]

where the correction factor \( K_w \) of nominal shear stress is

\[ K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \quad C = \frac{D}{d} \]

Spring force \( F \) and spring rate \( k \) are

\[ F = \frac{Gd^4}{8N_aD^2} \cdot f, \quad F = kf \]

Shear stress dependence on deflection is

\[ \tau_{xy,F} = K_k \cdot f = K_w \frac{d \cdot G}{\pi D^2 N_a} \cdot f \]
\[ K_k = K_w \frac{1}{\pi D^2 N_a} \]

The springs are generally pre-stressed with deflection \( f = f_{pre} \). The mechanism using the spring gives additional deflection \( f_{cam} \) and their sum is the maximum deflection

\[ f_{pre} = f_{max} \cdot f_{can} = \Delta h, \quad f_{max} = f_{min} + f_{can} \]

Shear stresses are by eq.(15)

\[ \tau = K_k f \Rightarrow \tau_{min} = K_k f_{min} = \tau_i, \quad \tau_{max} = K_k f_{max} \]

The mean and amplitude shear stresses are

\[ \tau_{mean} = \frac{1}{2}(\tau_{max} + \tau_{min}) \]
\[ \tau_{ampl} = \frac{1}{2}(\tau_{max} - \tau_{min}) \]

3.2.6 Properties evaluated by satisfaction functions

The following nine properties are selected as relevant.

3.2.6.1 Material cost of wire spring

Cost property is now wire material volume

\[ xx(1) = Volume = Length \cdot Area = \pi D N_a \frac{d}{4} d^2, \quad N_a = N_{tot} - 2 \]

Here \( N_{tot} \) is the total number of coils and \( N_a \) is the active number. It is 2 less due to the bent and ground end manufacturing.

![Fig. 4. Comparison of the three torsional mean stress vs. stress amplitude diagrams, Goodman, by Norton [2], Shigley-Mischke [3] and Spotts & al [4].](image-url)
3.2.6.3 Torsional safety factor estimation using Shigley-Mischke diagram

Torsional safety factor by Shigley- Mischke [3] model is
\[
\frac{1}{N_{\text{Misch}}} = \frac{\tau_{\text{w}}}{S_{\text{ey}}} + \frac{\tau_{\text{m}}}{S_{\text{us}}} , \quad xx(3) = N_{\text{Misch}} \tag{24}
\]

3.2.6.4 Torsional safety factor estimation using Spotts et al diagram

In this model by Spotts, Shoup and Hornberger [4] the yield strength in shear \( S_{\text{ys}} \) is defined by a different factor than by Goodman in Norton [2]. They also define a pulsating shear stress endurance \( \tau'_{\text{e}} \)
\[
S_{\text{ys}} = 0.51S_{\text{ut}} , \quad \tau'_{\text{e}} = 0.2S_{\text{ut}} \tag{25}
\]
These strength models are used in the model for calculating the safety factor
\[
\frac{K_w \tau_{\text{a}}}{S_{\text{ys}} - \tau_{\text{m}}} = \frac{1}{2} \frac{\tau_{\text{e}}'}{S_{\text{ys}} - \frac{1}{2} \tau_{\text{e}}'} , \quad xx(4) = N_{\text{spott}} \tag{26}
\]

3.2.6.5 Torsional safety factor estimation using an engineering method

This is based on Finnish standard procedures [6]. No shot peening is assumed. The allowed stress is calculated from static tensile strength \( R_{\text{m}} \) using conservative strength reduction factors due to loading severity and amplitude magnitude
\[
N_{\text{taual}} = \frac{\tau_{\text{all}}}{\tau_{\text{max}}} , \quad \tau_{\text{all}} = C_{\text{ampl}} \cdot C_{\text{life}} \cdot C_{\text{helical}} \cdot R_{\text{m}} \tag{27}
\]
\[
xx(5) = N_{\text{taual}}
\]
Here reduction factor for ensuring endurance at large amplitudes \( C_{\text{ampl}} = .8 \), factor ensuring long life is \( C_{\text{longlife}} = .9 \). Factor taking into account compression loading of a helical spring is \( C_{\text{helix}} = .31 \) and for tension loading 0.37. Now the more conservative option 0.31 is chosen.

3.2.6.6 Surge frequency

The surge frequency should be higher than the main operational frequency of the machine 10 Hz by [3]
\[
f_{\text{surge}} = \frac{1}{2\pi} \frac{d}{N_a D^3} \sqrt{\frac{G}{2\rho}}, \quad xx(7) = f_{\text{surge}} \tag{28}
\]

3.2.6.7 Spring rate within a desired range

\[
k = \frac{d^4 G}{8 D^3 N_a}, \quad xx(8) = k \tag{29}
\]

3.2.6.8 Impact on the spring end

The rotating moving mass of the cam hits an elastic steel body of a spring producing impact stress. Typical cams rotate with frequency \( f = 5 \cdots 10 \)Hz, period \( T = 1/f \) sec.Cam rise \( s = \Delta h \) takes place in about \( \Delta t = xT = 0.1 \cdot T \). Axial speed of rise is
\[
v_{0a} = \frac{\Delta h}{\Delta t} = \frac{s}{xT} = \frac{0.05}{0.1} \approx 3.5 \frac{m}{s} \tag{30}
\]
The impact factor is by Burr [12]
\[
V = \frac{1}{\beta} \left( \frac{1}{1 + \frac{1}{\beta}} \right), \quad \beta = \frac{m}{M} \tag{31}
\]
here \( \beta \) is mass ratio of object mass \( m \) and impactor mass \( M \). The following definitions are needed
\[
U = \frac{2e_{\text{e}}}{K_w}, \quad T_F = K_w T_{\text{nf}}, \quad T_{\text{nf}} = \frac{8 D}{\pi d^3} \tag{32}
\]
The static stresses in the spring are related as follows
\[
\tau_{xy} = T_F F, \quad \sigma_{X,F} = U \cdot \tau_{xy,F} \tag{33}
\]
The normal stress induced by impact is assumed as
\[
\sigma_i = \frac{\Delta h}{\Delta t} \sqrt{E \rho \cdot V} = \sigma_0 V \tag{34}
\]
The shear stress induced by impact is \( \tau_{\text{im}} \)
\[
\tau_{\text{im}} = \frac{d}{D} \sqrt{\frac{G}{E}}, \quad \sigma_{0,\text{im}} = H \cdot \sigma_0, \tag{35}
\]
\[
\tau_{\text{im}} = \tau_0 \cdot V, \quad \sigma_i = \sigma_0 \cdot V \tag{35}
\]

3.2.6.9 Fatigue life of spring by combining a Haigh diagram and the S-N diagram

This method of calculating fatigue life \( N_{\text{life}} \) combines the Haigh diagram of modified Goodman type and the S-N diagram according to Meyer [7].
\[
N_{\text{life}} = 10^A, \quad A = \log \left[ \frac{V_a V_e}{(1-V_{\text{m}})^2 \log(V_e/c)} \right] \tag{36}
\]
where three stress ratios are used
\[
V_a = \frac{\sigma_{\text{va}}}{R_{\text{m}}}, \quad V_m = \frac{\sigma_{\text{vm}}}{R_{\text{m}}}, \quad V_e = \frac{S_e}{R_{\text{m}}}, \quad c = 0.9 \tag{37}
\]

Fig. 5. Definitions of impacting loads causing shear and normal stress impacts. Angle of crack is often \( \alpha = 20^\circ \).
here $V_e$ is relative effective stress amplitude, $V_m$ is relative effective mean stress and $V_a$ is relative effective corrected fatigue strength. In these

$$\sigma_{vm} = \left( \sigma_m^2 + 3\sigma_m^2 \right)^{1/2}, \quad \sigma_{va} = \left( \sigma_a^2 + 3\sigma_a^2 \right)^{1/2} \quad (38)$$

The dynamic normal stress is dangerous in springs. It has been observed often that the cracks occur at angle $\alpha = 20^\circ$ and probably normal to max. principal stress. This means that the ratio of max. $\sigma_x$ stress to shear stress is about 2 giving angle $20^\circ$. Fig 4b

![Fig.6. The method of calculating fatigue lives of crack initiation time from normal mean stress and amplitude stress vs. S-N diagram.](image)

The ideal fatigue strength or the mean endurance limit of the rotating-bending specimens of steels can be calculated from static strength. The regression fit formula by Just [8] is used. Motivation is that it gives dependence on $Z$. Bellot and Gantois [9] give a formula for $Z$ in strong constructional steels with $R_m = 950\text{MPa}$. Fracture strain is obtained by a tensile test of material having a volume fraction $f$ of inclusions

$$e_f(Z) = \ln \left( \frac{1}{1 - Z} \right) = e_{ph}(f) \quad \Rightarrow \quad Z = 1 - e^{-e_f(Z)} \quad (39)$$

using this one obtains for the dependency of the ideal bending fatigue strength with zero mean stress on the static tensile strength with parameter $A = 1700$ [8]

$$\sigma_w = k_{Rm} R_m, \quad k_{Rm}(f) = 0.26 + 0.45 \cdot Z_f \quad (40)$$

Now presently a simple model is to describe $Z$ vs $f$

$$Z_f = Z_0 \frac{1}{1 + fA}, \quad k_{Rm}(0) = 0.5 = 0.26 + 0.45 \cdot Z_0. \quad (41)$$

The fully corrected fatigue strength is $S_e$

$$S_e = \frac{1}{K_f} \sigma_e = C_1 C_k \sigma_w \Rightarrow \quad S_e = C_1 C_p C_1 C_k C_s C_z C_m \sigma_w = C_{total} \sigma_w \quad (42)$$

Here $C_1$ and $C_1 = 1/K_f$ are fatigue strength reduction factors and $K_f$ is the notch effect factor. $C_a$ is shot peening factor. It can be even 1.2, $C_t$ is temperature factor, $C_r$ is reliability factor, $C_s$ is surface condition factor and $C_z$ is size factor. For beams of diameter $d(m)$ it is $C_z = 0.6085d^{-0.097}$. $8\text{mm} < d < 25\text{mm}$ by Hundal [9]. $C_m$ is corrosion factor. Shot peening factor is above unity and some factors are below. Now the total factor is about unity.

Eccentricity is obtained by a detailed calculation as, Fig.1

$$e_c = e_{CR} \cdot \frac{1}{2} D, \quad e_{CR} = 0.3 \quad (43)$$

Static bending stress at critical section at 1+1/8 turns is

$$\sigma_{xf} = e_{CR} \frac{1}{2} \frac{D}{R} \cdot F = \frac{16}{\pi} \frac{e_{CR} D}{\pi} \cdot F = S_{xf} \cdot F \quad (44)$$

Static shear stress vs. force $F$ is given by equation (45)

$$\tau_{xy,F} = K_w \tau_n = K_w T_{nf}, \quad F = T_F \cdot F \quad (45)$$

The resultant stresses are assumed to be sum of the continuously varying static stress due to force $F$ and the very short time impact stresses

$$\sigma_x = \sigma_{xy,F} + \sigma_0 V \quad (46)$$

Principal stress is dominant in activating fatigue crack initiation

$$\sigma_1 = \frac{1}{2} \sigma_x + \sqrt{\left( \frac{1}{2} \sigma_x \right)^2 + \tau_{xy}^2} \quad (47)$$

The maximum and minimum values of stress depend on load force

$$F_{\text{max}} = k_f F_{\text{max}}, \quad F_{\text{min}} = k_f F_{\text{min}} \Rightarrow \quad (48)$$

Mean values of principal stress and amplitude are

$$\sigma_{1,\text{mean}} = \frac{1}{2} \left( \sigma_{1,\text{max}} + \sigma_{1,\text{min}} \right) \quad (49)$$

$$\sigma_{1,\text{ampl}} = \frac{1}{2} \left( \sigma_{1,\text{max}} - \sigma_{1,\text{min}} \right) \quad (50)$$

Now at principal direction the shear stress is zero

$$\sigma_{vm} = \sigma_{1,\text{mean}}, \quad \sigma_{va} = \sigma_{1,\text{ampl}} \quad (50)$$

### 3.2.6.10 Normalised height needed to fit the spring into given space

Free length $L_0$ for spring is sum of fully compressed length $L_c$, reserve deflection $S_a$ and usable deflection $f$. Using DIN 2095 [10] data fit gives for factor $x$

$$x = 0.00115(C - 4)^2 + 0.02(C - 4) + 0.1, \quad d \geq 0.8\text{mm} \quad (51)$$

Free length is

$$L_0 = L_c + S_a + f = N_1 d + xdN_a + f, \quad N_1 = N_a + 2 \quad (52)$$

Relative height is

$$\text{height} = \frac{L_{0,\text{calc}}}{L_{0,\text{given}}} = \left( N_1 [1 + x] + 2 \right) + \frac{L}{L_{0,\text{calc}}} \Rightarrow 1 \pm \Delta = xx(9) \quad (53)$$

### 3.2.7 Satisfaction function for each property

These are defined at four points at Table 1.
4. RESULTS

4.1 Results of Optimum Design

Results are shown in Tables 3 and 4. Geometry 564: 
\[ d(5) = 0.014, D(6) = 0.12, N(4) = 7 \]

Table 2: Optimisation results by search algorithm. Optimal values are shown bold. Geometry code 564

| d(lit) (m) | 0.008 | 0.009 | 0.010 | 0.012 | 0.014 | 0.016 |
| D(cm) (m) | 0.070 | 0.080 | 0.090 | 0.100 | 0.110 | 0.120 |
| N_{oef}(N_{tot}) | 4 | 5 | 6 | 7 | 8 | - |

Table 3: Results of optimisation. Geometry code 564

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<th>Case</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
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<th>R6</th>
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</tr>
</tbody>
</table>

Here the spring rate \( k = 44000N/m, e_{er} = \text{eccentricity} /0.5D, V = 4 \) is impact factor. Safety factor \( N_{aual} \) is for no shot peening, P(G) is total satisfaction. Angle of principal stress to axis of the wire is \( \alpha = 28^\circ \) which is larger than measured \( 20^\circ \).

From Table 3 it may be seen the following trends with rather high impact \( V = 4 \):
- Increase in eccentricity at case R1 from \( e_{er} = 0 \) to case R2 with \( e_{er} = 0.3 \) at no inclusions \( f = 0 \) decreases

\[ \log(N_{life}) = A \text{ from } 11.6 \text{ to } 8 \text{ causing fatigue life to decrease by a factor } 1/4000. \]
- Increase of inclusions from \( f = 0 \) to \( f = 1% \) decreases \( A \) from 8 (R1) to 5.4 (R3) by a factor a 1/400
- Goodman and Shigley-Mischke models take into account the effect of shot peening, but not Spotts model

At well designed springs with no eccentricity and no inclusions \( e_{er} =0, f =0 \) and no wearing impacts shot peening increases safety factors \( N_{godm} \) from 1 (R2) to 1.5(R6), or by a factor 1.5 \( N_{misch} \) from 1.33 (R2) to 1.65 (R6) or by a factor 1.24.
- At the case R6 maximum total satisfaction is 0.03 when there is no eccentricity, shot peening protection is preserved even with high impact \( V = 4 \) loading, and inclusion content is minimal using high quality steel. This is the goal recommended by this optimum redesign approach.
- The most optimal case is obtained from case R6 by setting \( V = 0 \) and \( e_{er}=0, f=0 \). Then one obtains \( A =13.2 \) and angle \( \alpha = 45^\circ \) corresponding to pure shear loading failure with no bending stress
- When at case R1 \( V \) is varied \( V = 0..20 \), the angle \( \alpha \) between x-axis and largest principal stress varied as

\[ \text{With no impacts } V = 0, \alpha = 45^\circ, \text{ shear dominates} \]
\[ \text{With medium impacts } V = 4, \alpha = 28^\circ \]
\[ \text{With very high impacts } V = 20, \alpha = 11^\circ, \text{Bending mostly} \]

And example of the worst case R3 is shown in Table 4. Low factors of safety an low life time are due to adverse factors: maxima eccentricity \( e_{er}=0.3 \), high inclusion content \( f = 0.01 \), high impact loading factor \( V = 4 \) and or no shot peening, Isp=1, at the critical section having high bending stresses. These results agree with results in [14].

Table 4: Details for the worst case R3. Code 564.

<table>
<thead>
<tr>
<th>IG</th>
<th>1 Cost</th>
<th>2 N_{godm}</th>
<th>3 N_{misch}</th>
<th>4 N_{spott}</th>
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<tr>
<td>xx(IG)</td>
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<tr>
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<td>0.685</td>
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<td>6 A_{aish}</td>
<td>7 f_{surge}</td>
<td>8 k_{spring}</td>
</tr>
<tr>
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<td>5.4</td>
<td>231</td>
<td>44000</td>
</tr>
<tr>
<td>p(IG)</td>
<td>0.37</td>
<td>0.1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

4.2 FEM analysis results

The MSC Nastran FEM [13] program is used. Geometry is shown in Figure 7. The dimensions in the case study using FEM were \( d =10\text{mm}, D =100\text{mm}, \text{total number of coils } N_{tot} = 8 \). The end were not ground or bent, but the load was applied by an even distribution of point forces for the \( 270^\circ \) arc. Effectively 1 coil was fixed now.
The analytical spring rate was $k = 14000$N/m and FEM gave the same.

Symmetry of loading affects von Mises stress

VM = 325 MPa for symmetric loading

VM = 263 MPa for non-symmetric 270° arc.

Among the many natural frequency modes studied the first which caused upper turn to impact each other had natural frequency 2055 Hz.

5. CONCLUSIONS

The following conclusions can be drawn

- Standards assume in helical springs only torsional stresses with no bending with impacts which arise due to eccentric load force application
- Impact loading increases the static torsion and but more bending stresses. life time predictions decrease by many decades. The predicted angle of the largest principal stress relative to axis is close to the observed angle.
- Shot peening gives protection by compressive surface residual stresses. But notable wear between coils can delete it from critical areas.
- Inclusions at critical bent surface areas reduce lifetimes notably
- Optimal design dimensioning guarantees satisfactory long reliable service life
- FEM analyses reveal stress gradients agreeing with the analytical results.
- Further research to solve the definite cause of the impacts is needed. FEM with wearing out of elements may be useful. One interesting cause to be studied is whether the natural mode causing upper coil impacts is really activated.

REFERENCES